Theorem proof

In mathematics, the ratio test is a test (or "criterion") for the convergence of a series

where each term is a real or complex number and an is nonzero when n is large.

**Theorem( d'Alembert's "criterion")**



The ratio test states that:

if *L* < 1 then the series [converges absolutely](https://en.wikipedia.org/wiki/Absolute_convergence);

if *L* > 1 then the series [diverges](https://en.wikipedia.org/wiki/Divergent_series);

**Proof**:

Suppose that {\displaystyle L=\lim \_{n\to \infty }\left|{\frac {a\_{n+1}}{a\_{n}}}\right|<1} .

We can then show that the series converges absolutely by showing that its terms will eventually become less than those of a certain convergent [geometric series](https://en.wikipedia.org/wiki/Geometric_series).

To do this, consider a real number *r such that*  {\displaystyle L<r<1}

This implies that  {\displaystyle |a\_{n+1}|<r|a\_{n}|}for sufficiently large *n*; say, for all *n* greater than *N*.

Hence  {\displaystyle |a\_{n+i}|<r^{i}|a\_{n}|}  for each *n* > *N* and *i* > 0, and so



That is, the series converges absolutely.

On the other hand, if *L* > 1, then  {\displaystyle |a\_{n+1}|>|a\_{n}|} for sufficiently large *n*, so that the limit of the summands is non-zero. Hence the series diverges.

Theorem is proved